CHAPTER 7

Variable Conversion

Chapter Outline

Measurement and Instrumentation. http://dx.doi.org/10.1016/B978-0-12-800884-3.00007-1

7.1 Introduction

The techniques for data acquisition as discussed in the last chapter, and also those for displaying or recording measurement data as discussed later in Chapter 9, are only directly applicable for measurement sensors that have an output in the form of a varying voltage signal. Unfortunately, there are many measurement sensors that do not have an output in this convenient voltage form, but rather have an output in some other form. Such other forms include translational displacements and changes in various nonvoltage electrical parameters such as resistance, inductance, capacitance, and current. In some cases, the output may alternatively take the form of variations in the phase or frequency of an alternating current electrical signal.

We therefore need to have a means of converting sensor outputs that are initially in a nonvoltage form into the more convenient form. This can be achieved by inserting various types of variable conversion element into the measurement system. We will consider these in detail in this chapter. First, we will see that bridge circuits are a particularly important type of variable conversion element, and these will be covered in some detail. Following this, we will look at various alternative techniques for transducing the outputs of a measurement sensor into a form that is more readily measured.

7.2 Bridge Circuits

Bridge circuits are used very commonly as a variable conversion element in measurement systems and produce an output in the form of a voltage level that changes as the measured physical quantity changes. They provide an accurate method of measuring resistance, inductance, and capacitance values, and enable the detection of very small changes in these quantities about a nominal value. They are of immense importance in the measurement system technology because so many transducers measuring physical quantities have an output that is expressed as a change in resistance, inductance, or capacitance. The displacement-measuring strain gauge, which has a varying resistance output, is but one example of this class of transducers. Normally, excitation of the bridge is by a DC voltage for resistance measurement and by an AC voltage for inductance or capacitance measurement. Both null and deflection types of bridge exist, and, in a like manner to instruments in general, null types are mainly employed for calibration purposes and deflection types are used within closed-loop automatic control schemes.

7.2.1 Null-Type, DC Bridge (Wheatstone Bridge)

A null-type bridge with DC excitation, commonly known as a Wheatstone Bridge, has the form shown in Figure 7.1. The four arms of the bridge consist of the unknown resistance

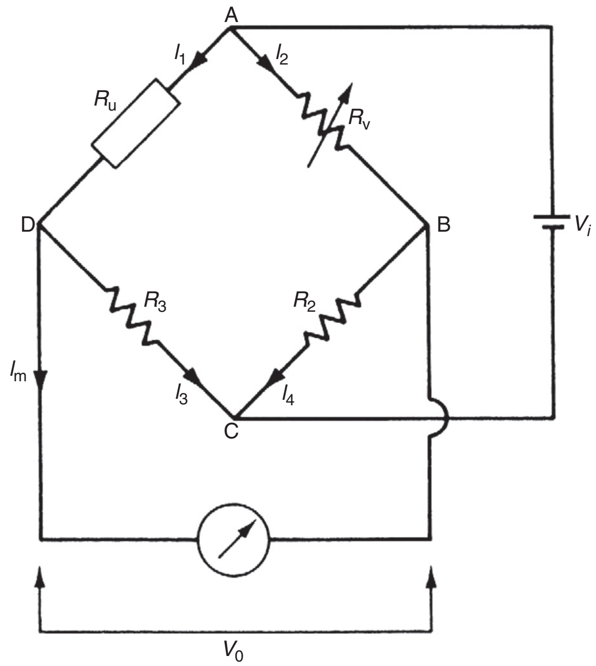

Figure 7.1
Wheatstone bridge.

R_u, two equal value resistors R_2 and R_3, and a variable resistor R_v (usually a decade resistance box). A DC voltage V_i is applied across the points AC and the resistance R_v is varied until the voltage measured across points BD is zero. This null point is usually measured with a high-sensitivity galvanometer.

To analyze the Wheatstone Bridge, define the current flowing in each arm to be $I_1 \ldots I_4$ as shown in Figure 7.1. Normally, if a high-impedance voltage-measuring instrument is used, the current I_m drawn by the measuring instrument will be very small and can be approximated to zero. If this assumption is made, then, for $I_m = 0$: $I_1 = I_3$ and $I_2 = I_4$.

Looking at path ADC, we have a voltage V_i applied across a resistance $R_u + R_3$ and by Ohm's law:

$$I_1 = \frac{V_i}{R_u + R_3}$$

Similarly, for path ABC:

$$I_2 = \frac{V_i}{R_v + R_2}$$

Now we can calculate the voltage drop across AD and AB:

$$V_{AD} = I_1 R_v = \frac{V_i R_u}{R_u + R_3}; \quad V_{AB} = I_2 R_v = \frac{V_i R_v}{R_v + R_2}$$

By the principle of superposition, $V_0 = V_{BD} = V_{BA} + V_{AD} = -V_{AB} + V_{AD}$

Thus:

$$V_0 = -\frac{V_i R_v}{R_v + R_2} + \frac{V_i R_u}{R_u + R_3} \tag{7.1}$$

At the null point $V_0 = 0$, so

$$\frac{R_u}{R_u + R_3} = \frac{R_v}{R_v + R_2}$$

Inverting both sides:

$$\frac{R_u + R_3}{R_u} = \frac{R_v + R_2}{R_v} \quad \text{i.e.,} \quad \frac{R_3}{R_u} = \frac{R_2}{R_v} \quad \text{or} \quad R_u = \frac{R_3 R_v}{R_2} \tag{7.2}$$

Thus, if $R_2 = R_3$, then $R_u = R_v$. As R_v is an accurately known value because it is derived from a variable decade resistance box, this means that R_u is also accurately known.

A null-type bridge is somewhat tedious to use since careful adjustment of the variable resistance is needed to get exactly to the null point. However, it provides a highly accurate measurement of resistance, leading to this being the preferred type when sensors are being calibrated.

Example 7.1

A null-type Wheatstone bridge is used to accurately measure the resistance of a platinum resistance thermometer during a calibration procedure. The circuit shown in Figure 7.1 is used, in which the known fixed resistance values are given by $R_2 = 98.3\ \Omega$ and $R_3 = 102.2\ \Omega$. The thermometer is inserted in the circuit as R_u and then the variable resistance box R_v is adjusted until the bridge output voltage V_0 goes to zero. At this balance point, the value of R_v is 95.7 Ω. Calculate the resistance of the thermometer.

Solution

At the balance point, the resistance values are related according to Eqn (7.2):

$$R_u = \frac{R_3 R_v}{R_2}.$$

Substituting the resistance values into Eqn (7.2):

$$R_u = \frac{102.2 \times 95.7}{98.3} = 99.5\ \Omega$$

Thus, the resistance of the thermometer is 99.5 Ω.

■

7.2.2 Deflection-Type DC Bridge

A deflection-type bridge with DC excitation is shown in Figure 7.2. This differs from the Wheatstone bridge mainly in that the variable resistance R_v is replaced by a fixed resistance R_1 of the same value as the nominal value of the unknown resistance R_u. As the resistance R_u changes, the output voltage V_0 varies, and this relationship between V_0 and R_u must be calculated.

This relationship is simplified if we again assume that a high-impedance voltage-measuring instrument is used and the current drawn by it, I_m, can be approximated to zero. (The case when this assumption does not hold is covered later in this section.) The

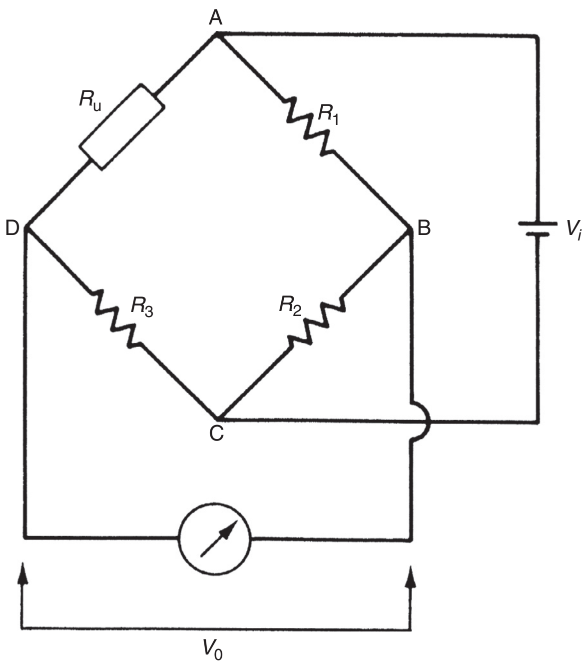

Figure 7.2
Deflection-type DC bridge.

analysis is then exactly the same as for the preceding example of the Wheatstone bridge, except that R_v is replaced by R_1. Thus, from Eqn (7.1), we have:

$$V_0 = V_i\left(\frac{R_u}{R_u + R_3} - \frac{R_1}{R_1 + R_2}\right) \tag{7.3}$$

When R_u is at its nominal value, that is, for $R_u = R_1$, it is clear that $V_0 = 0$ (since $R_2 = R_3$). For other values of R_u, V_0 has negative and positive values that vary in a nonlinear way with R_u.

The deflection-type bridge is somewhat easier to use than a null-type bridge since the output measurement is given directly in the form of a voltage measurement. However, its measurement accuracy is not as good as that of a null-type bridge. In spite of its inferior accuracy, ease of use means that it is the preferred form of bridge in most general measurement situations unless the greater accuracy of a null-type bridge is absolutely necessary.

Example 7.2

A certain type of pressure transducer, designed to measure pressures in the range of 0–10 bar, consists of a diaphragm with a strain gauge cemented to it to detect diaphragm deflections. The strain gauge has a nominal resistance of 120 Ω and forms one arm of a Wheatstone bridge circuit, with the other three arms each having a resistance of 120 Ω. The bridge output is measured by an instrument whose input impedance can be assumed infinite. If, in order to limit heating effects, the maximum permissible gauge current is 30 mA, calculate the maximum permissible bridge excitation voltage. If the sensitivity of the strain gauge is 338 mΩ/bar and the maximum bridge excitation voltage is used, calculate the bridge output voltage when measuring a pressure of 10 bar.

Solution

This is the type of bridge circuit shown in Figure 7.2 in which the components have the following values:

$$R_1 = R_2 = R_3 = 120\ \Omega$$

Defining I_1 to be the current flowing in path ADC of the bridge, we can write:

$$V_i = I_1(R_u + R_3)$$

At balance, $R_u = 120$ and the maximum value allowable for I_1 is 0.03 A.

Hence, $V_i = 0.03(120 + 120) = 7.2$ V.

Thus, the maximum bridge excitation voltage allowable is 7.2 V.

For a pressure of 10 bar applied, the resistance change is 3.38 Ω, that is, R_u is then equal to 123.38 Ω.

Applying Eqn (7.3), we can write:

$$V_0 = V_i\left(\frac{R_u}{R_u + R_3} - \frac{R_1}{R_1 + R_2}\right) = 7.2\left(\frac{123.38}{243.38} - \frac{120}{240}\right) = 50 \text{ mV}$$

Thus, if the maximum permissible bridge excitation voltage is used, the output voltage is 50 mV when a pressure of 10 bar is measured.

■

The nonlinear relationship between the output reading and the measured quantity exhibited by Eqn (7.3) is inconvenient and does not conform with the normal requirement for a linear input–output relationship. The method of coping with this nonlinearity varies according to the form of primary transducer involved in the measurement system.

One special case is where the change in the unknown resistance R_u is typically small compared with the nominal value of R_u. If we calculate the new voltage V_0' when the resistance R_u in Eqn (7.3) changes by an amount δR_u, we have:

$$V_0' = V_i\left(\frac{R_u + \delta R_u}{R_u + \delta R_u + R_3} - \frac{R_1}{R_1 + R_2}\right) \tag{7.4}$$

The change of voltage output is therefore given by:

$$\delta V_0 = V_0' - V_0 = \frac{V_i \delta R_u}{R_u + \delta R_u + R_3}$$

If $\delta R_u << R_u$, then the following linear relationship is obtained:

$$\frac{\delta V_0}{\delta R_u} = \frac{V_i}{R_u + R_3} \tag{7.5}$$

This expression describes the measurement sensitivity of the bridge. Such an approximation to make the relationship linear is valid for transducers such as strain gauges where the typical changes of resistance with strain are very small compared with the nominal gauge resistance.

However, many instruments that are inherently linear themselves at least over a limited measurement range, such as resistance thermometers, exhibit large changes in output as the input quantity changes, and the approximation of Eqn (7.5) cannot be applied. In such cases, specific action must be taken to improve linearity in the relationship between the bridge output voltage and the measured quantity. One common solution to this problem is to make the values of the resistances R_2 and R_3 at least 10 times those of R_1 and R_u (nominal). The effect of this is best observed by looking at a numerical example.

Consider a platinum resistance thermometer with a range of 0°–50 °C, whose resistance at 0 °C is 500 Ω and whose resistance varies with temperature at a rate of 4 Ω/°C. Over this range of measurement, the output characteristic of the thermometer itself is nearly perfectly linear. (N.B.: The subject of resistance thermometers is discussed further in Chapter 14.)

Taking first the case where $R_1 = R_2 = R_3 = 500\ \Omega$ and $V_i = 10$ V, and applying Eqn (7.3):

$$\text{At } 0\ ^\circ\text{C} \;;\; V_0 = 0$$
$$\text{At } 25\ ^\circ\text{C} \;;\; R_u = 600\ \Omega \quad \text{and} \quad V_0 = 10\left(\frac{600}{1100} - \frac{500}{1000}\right) = 0.455\ \text{V}$$
$$\text{At } 50\ ^\circ\text{C} \;;\; R_u = 700\ \Omega \quad \text{and} \quad V_0 = 10\left(\frac{700}{1200} - \frac{500}{1000}\right) = 0.833\ \text{V}$$

This relationship between V_0 and R_u is plotted as curve (A) in Figure 7.3 and the nonlinearity is apparent. Inspection of the manner in which the output voltage V_0 changes for equal steps of temperature change also clearly demonstrates the nonlinearity.

For the temperature change from 0 to 25 °C, the change in V_0 is (0.455 − 0) =0.455 V.

For the temperature change from 25 to 50 °C, the change in V_0 is (0.833 − 0.455) = 0.378 V.

If the relationship was linear, the change in V_0 for the 25–50 °C temperature step would also be 0.455 V, giving a value for V_0 of 0.910 V at 50 °C.

Now take the case where $R_1 = 500\ \Omega$ but $R_2 = R_3 = 5000\ \Omega$ and let $V_i = 26.1$ V:

$$\text{At } 0\ ^\circ\text{C} \;;\; V_0 = 0$$
$$\text{At } 25\ ^\circ\text{C} \;;\; R_u = 600\ \Omega \quad \text{and} \quad V_0 = 26.1\left(\frac{600}{5600} - \frac{500}{5500}\right) = 0.424\ \text{V}$$
$$\text{At } 50\ ^\circ\text{C} \;;\; R_u = 700\ \Omega \quad \text{and} \quad V_0 = 26.1\left(\frac{700}{5700} - \frac{500}{5500}\right) = 0.833\ \text{V}$$

This relationship is shown as curve (B) in Figure 7.3 and a considerable improvement in linearity is achieved. This is more apparent if the differences in values for V_0 over the two temperature steps are inspected.

From 0 to 25 °C, the change in V_0 is 0.424 V.

From 25 to 50 °C, the change in V_0 is 0.409 V.

The changes in V_0 over the two temperature steps are much closer to being equal than before, demonstrating the improvement in linearity. However, in increasing the values

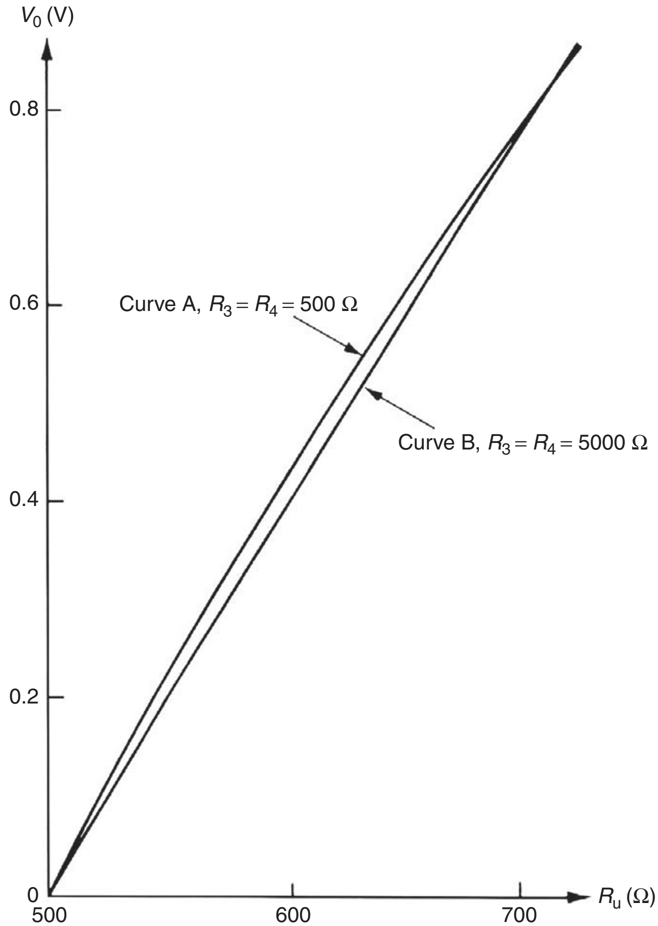

Figure 7.3
Linearization of bridge circuit characteristic.

of R_2 and R_3, it was also necessary to increase the excitation voltage from 10 to 26.1 V to obtain the same output levels. In practical applications, V_i would normally be set at the maximum level consistent with the limitation of the effect of circuit heating in order to maximize the measurement sensitivity ($V_0/\delta R_u$ relationship). It would therefore not be possible to increase V_i further if R_2 and R_3 were increased, and the general effect of such an increase in R_2 and R_3 is thus a decrease in the sensitivity of the measurement system.

The importance of this inherent nonlinearity in the bridge output relationship is greatly diminished if the primary transducer and bridge circuit are incorporated as elements within an intelligent instrument. In that case, digital computation is applied to produce an output in terms of the measured quantity that automatically compensates for the nonlinearity in the bridge circuit.

Case where the current drawn by measuring instrument is not negligible

For various reasons, it is not always possible to meet the condition that the impedance of the instrument measuring the bridge output voltage is sufficiently large for the current drawn by it to be negligible. Wherever the measurement current is not negligible, an alternative relationship between the bridge input and output must be derived that takes the current drawn by the measuring instrument into account.

Thévenin's theorem is again a useful tool for this purpose. Replacing the voltage source V_i in Figure 7.4(a) by a zero internal resistance produces the circuit shown in Figure 7.4(b), or the equivalent representation shown in Figure 7.4(c). It is apparent from Figure 7.4(c)

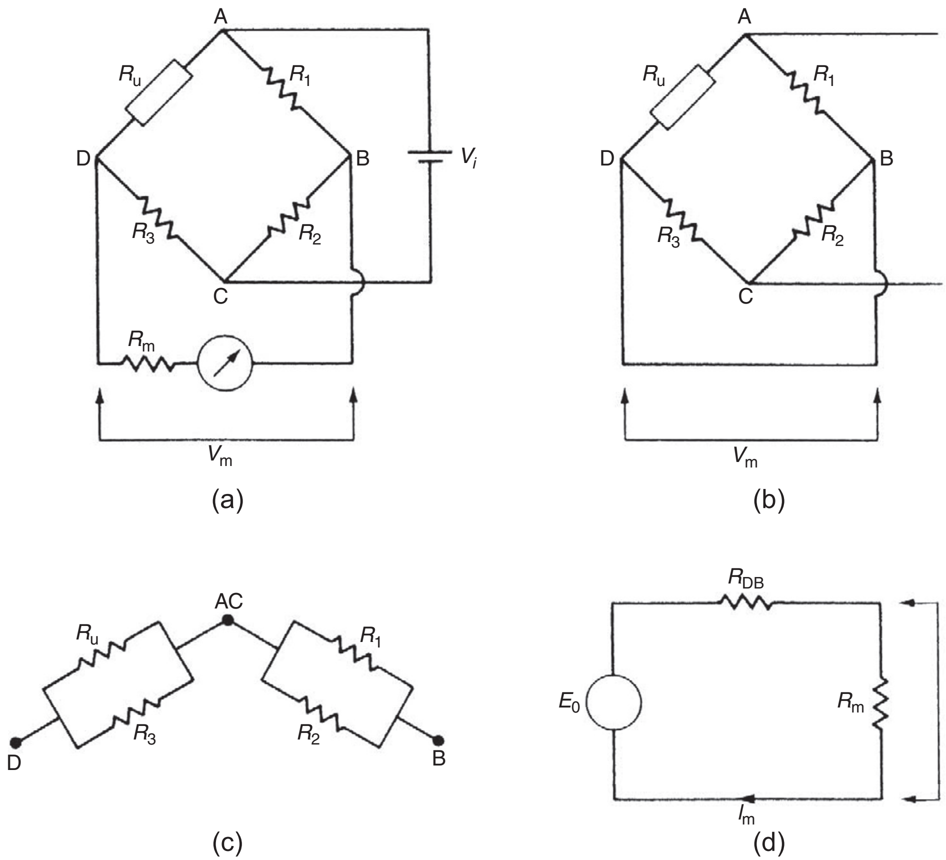

Figure 7.4
(a) A bridge circuit; (b) equivalent circuit by Thévenin's theorem; (c) alternative representation; (d) equivalent circuit for alternative representation.

that the equivalent circuit resistance consists of a pair of parallel resistors R_u and R_3 in series with the parallel resistor pair R_1 and R_2. Thus, R_{DB} is given by:

$$R_{DB} = \frac{R_1 R_2}{R_1 + R_2} + \frac{R_u R_3}{R_u + R_3} \tag{7.6}$$

The equivalent circuit derived via Thévenin's theorem with the resistance R_m of the measuring instrument connected across the output is shown in Figure 7.4(d). The open-circuit voltage across DB, E_0, is the output voltage calculated earlier (Eqn (7.3)) for the case of $R_m = 0$:

$$E_0 = V_i \left(\frac{R_u}{R_u + R_3} - \frac{R_1}{R_1 + R_2} \right) \tag{7.7}$$

If the current flowing is I_m when the measuring instrument of resistance R_m is connected across DB, then, by Ohm's law, I_m is given by:

$$I_m = \frac{E_0}{R_{DB} + R_m} \tag{7.8}$$

If V_m is the voltage measured across R_m, then, again by Ohm's law:

$$V_m = I_m R_m = \frac{E_0 R_m}{R_{DB} + R_m} \tag{7.9}$$

Substituting for E_0 and R_{DB} in Eqn (7.9), using the relationships developed in Eqns (7.6) and (7.7), we obtain:

$$V_m = \frac{V_i [R_u/(R_u + R_3) - R_1/(R_1 + R_2)] R_m}{R_1 R_2/(R_1 + R_2) + R_u R_3/(R_u + R_3) + R_m}$$

Simplifying:

$$V_m = \frac{V_i R_m (R_u R_2 - R_1 R_3)}{R_1 R_2 (R_u + R_3) + R_u R_3 (R_1 + R_2) + R_m (R_1 + R_2)(R_u + R_3)} \tag{7.10}$$

Example 7.3

A bridge circuit, as shown in Figure 7.5, is used to measure the value of the unknown resistance R_u of a strain gauge of nominal value 500 Ω. The output voltage measured across points DB in the bridge is measured by a voltmeter. Calculate the measurement sensitivity in volts/ohm change in R_u if

(a) the resistance R_m of the measuring instrument is neglected and
(b) account is taken of the value of R_m.

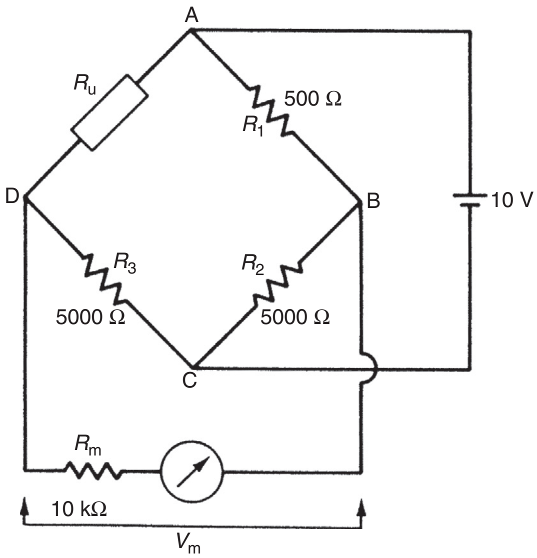

Figure 7.5
Bridge circuit for Example 7.2.

Solution

For $R_u = 500\ \Omega$, $V_m = 0$.

To determine sensitivity, calculate V_m for $R_u = 501\ \Omega$.

(a) Applying Eqn (7.3):

$$V_m = V_i\left(\frac{R_u}{R_u + R_3} - \frac{R_1}{R_1 + R_2}\right)$$

Substituting in values:

$$V_m = 10\left(\frac{501}{1001} - \frac{500}{1000}\right) = 5.00\ \text{mV}$$

Thus, if the resistance of the measuring circuit is neglected, the measurement sensitivity is 5.00 mV per ohm change in R_u.

(b) Applying Eqn (7.10) and substituting in values:

$$V_m = \frac{10 \times 10^4 \times 500(501 - 500)}{500^2(1001) + 500 \times 501(1000) + 10^4 \times 1000 \times 1001} = 4.76\ \text{mV}$$

Thus, if proper account is taken of the 10 kΩ value of the resistance of R_m, the true measurement sensitivity is shown to be 4.76 mV per ohm change in R_u.

7.2.3 Error Analysis

In the application of bridge circuits, the contribution of component value tolerances to total measurement system accuracy limits must be clearly understood. The analysis below applies to a null-type (Wheatstone) bridge, but similar principles can be applied for a deflection-type bridge. The maximum measurement error is determined by first finding the value of R_u in Eqn (7.2) with each parameter in the equation set at that limit of its tolerance which produces the maximum value of R_u. Similarly, the minimum possible value of R_u is calculated, and the required error band is then the span between these maximum and minimum values.

Example 7.4

In the Wheatstone bridge circuit of Figure 7.1, R_v is a decade resistance box with a specified inaccuracy of $\pm 0.2\%$ and $R_2 = R_3 = 500\ \Omega \pm 0.1\%$. If the value of R_v at the null position is $520.4\ \Omega$, determine the error band for R_u expressed as a percentage of its nominal value.

Solution

Applying Eqn (7.2) with $R_v = 520.4\ \Omega + 0.2\% = 521.44\ \Omega$, $R_3 = 5000\ \Omega + 0.1\% = 5005\ \Omega$, $R_2 = 5000\ \Omega - 0.1\% = 4995\ \Omega$, we get:

$$R_v = \frac{521.44 \times 5005}{4995} = 522.48\ \Omega\ (= +0.4\%)$$

Applying Eqn (7.2) with $R_v = 520.4\ \Omega - 0.2\% = 519.36\ \Omega$, $R_3 = 5000\ \Omega - 0.1\% = 4995\ \Omega$, $R_2 = 5000\ \Omega + 0.1\% = 5005\ \Omega$, we get:

$$R_v = \frac{519.36 \times 4995}{5005} = 518.32\ \Omega\ (= -0.4\%)$$

Thus, the error band for R_u is $\pm 0.4\%$.

The cumulative effect of errors in individual bridge circuit components is clearly seen. Although the maximum error in any one component is $\pm 0.2\%$, the possible error in the measured value of R_u is $\pm 0.4\%$. Such a magnitude of error is often not acceptable, and special measures are taken to overcome the introduction of error by component value tolerances. One such practical measure is the introduction of apex balancing. This is one of many methods of bridge balancing that all produce a similar result.

Apex balancing

One form of apex balancing consists of placing an additional variable resistor R_5 at junction C between the resistances R_2 and R_3, and applying the excitation voltage V_i to the wiper of this variable resistance, as shown in Figure 7.6.

For calibration purposes, R_u and R_v are replaced by two equal resistances whose values are accurately known, and R_5 is varied until the output voltage V_0 is zero. At this point, if the portions of resistance on either side of the wiper on R_5 are R_6 and R_7 (such that $R_5 = R_6 + R_7$), we can write:

$$R_3 + R_6 = R_2 + R_7$$

We have thus eliminated any source of error due to the tolerance in the value of R_2 and R_3, and the error in the measured value of R_u depends only on the accuracy of one component, the decade resistance box R_v.

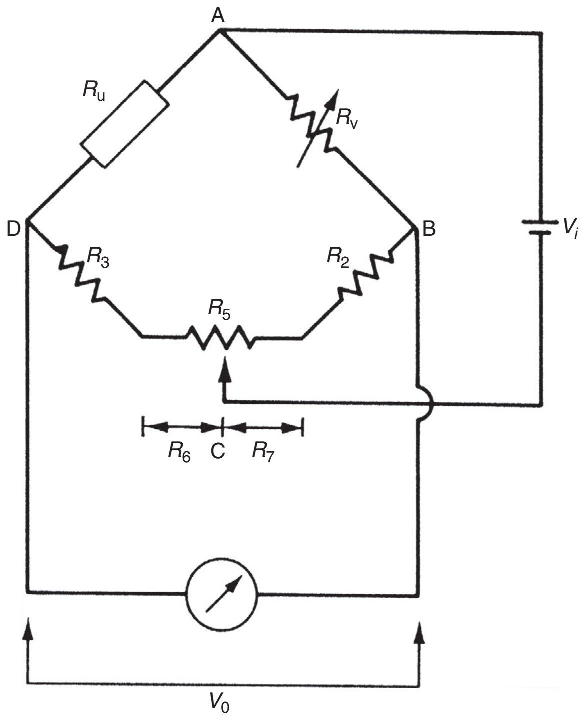

Figure 7.6
Apex balancing.

Example 7.5

A potentiometer R_5 is put into the apex of the bridge shown in Figure 7.6 to balance the circuit. The bridge components have the following values:

$$R_u = 500\ \Omega, \quad R_v = 500\ \Omega, \quad R_2 = 515\ \Omega, \quad R_3 = 480\ \Omega, \quad R_5 = 100\ \Omega$$

Determine the required value of the resistances R_6 and R_7 of the parts of the potentiometer track on either side of the slider in order to balance the bridge and compensate for the unequal values of R_2 and R_3.

Solution

For balance, $R_2 + R_7 = R_3 + R_6$; hence, $515 + R_7 = 480 + R_6$.

Also, because R_6 and R_7 are the two parts of the potentiometer track R_5 whose resistance is 100 Ω:

$R_6 + R_7 = 100$; thus $515 + R_7 = 480 + (100 - R_7)$; that is, $2R_7 = 580 - 515 = 65$.

Thus, $R_7 = 32.5$; hence, $R_6 = 100 - 32.5 = 67.5\ \Omega$.

7.2.4 AC Bridges

Bridges with AC excitation are used to measure unknown impedances (capacitances and inductances). Both null and deflection types exist. As for DC bridges, null-types are more accurate but also more tedious to use. Therefore, null types are normally reserved for use in calibration duties and any other application where very high measurement accuracy is required. Otherwise, in all other general applications, deflection types are preferred.

Null-type impedance bridge

A typical null-type impedance bridge is shown in Figure 7.7. The null point can be conveniently detected by monitoring the output with a pair of headphones connected via an operational amplifier across the points BD. This is a much cheaper method of null detection than the application of an expensive galvanometer that is required for a DC Wheatstone bridge.

Referring to Figure 7.7, at the null point, $I_1 R_1 = I_2 R_2$; $I_1 Z_u = I_2 Z_v$.

Thus,

$$Z_u = \frac{Z_v R_1}{R_2} \tag{7.11}$$

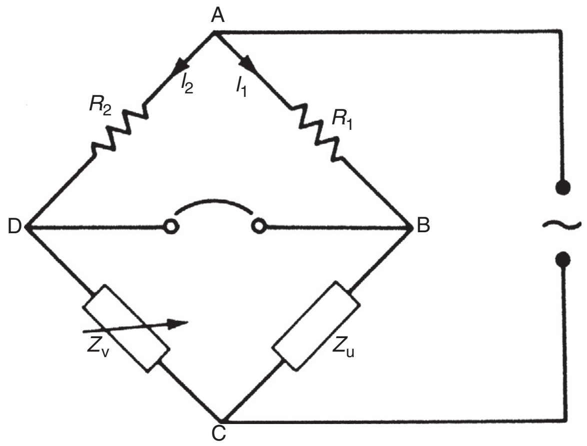

Figure 7.7
Null-type impedance bridge.

If Z_u is capacitive, that is, $Z_u = 1/j\omega C_u$, then Z_v must consist of a variable capacitance box, which is readily available. If Z_u is inductive, then $Z_u = R_u + j\omega L_u$.

Note that the expression for Z_u as an inductive impedance has a resistive term in it because it is impossible to realize a pure inductor. An inductor coil always has a resistive component, though this is made as small as possible by designing the coil to have a high Q factor (Q factor is the ratio inductance/resistance). Therefore, Z_v must consist of a variable resistance box and a variable inductance box. However, the latter are not readily available because it is difficult and hence expensive to manufacture a set of fixed-value inductors to make up a variable-inductance box. For this reason, an alternative kind of null-type bridge circuit, known as the *Maxwell bridge*, is commonly used to measure unknown inductances.

Example 7.6

A null-type impedance bridge is used to accurately measure the capacitance of a capacitive pressure sensor during a calibration procedure. The circuit shown in Figure 7.7 is used, in which the known fixed resistance values are given by $R_1 = 491.7\ \Omega$ and $R_2 = 483.2\ \Omega$. The pressure sensor is inserted in the circuit as Z_u and an accurate variable capacitor box with capacitance C_v is used for Z_v. The capacitor box is adjusted until the bridge output voltage goes to zero. At this balance point, the value of C_v is 103.7 pF. Calculate the capacitance of the pressure sensor.

Solution

At the balance point, the bridge circuit components are related by Eqn (7.11), $Z_u = \frac{Z_v R_1}{R_2}$, where $Z_u = 1/j\omega C_u$ and $Z_v = 1/j\omega C_v$.

Substituting for Z_u and Z_v in Eqn (7.11) gives:

$$C_u = \frac{C_v R_2}{R_1}$$

Substituting the resistance values into this equation for C_u gives:

$$C_u = \frac{103.7 \times 483.2}{491.7} = 101.4 \text{ pF.}$$

Thus, the capacitance of the pressure sensor is 101.4 pF.

Maxwell bridge

A Maxwell bridge is shown in Figure 7.8. The requirement for a variable inductance box is avoided by introducing instead a second variable resistance. The circuit requires one standard fixed-value capacitor, two variable resistance boxes, and one standard fixed-value resistor, all of which are components that are readily available and inexpensive. Referring to Figure 7.8, we have at the null-output point:

$$I_1 Z_{AD} = I_2 Z_{AB} \quad ; \quad I_1 Z_{DC} = I_2 Z_{BC}$$

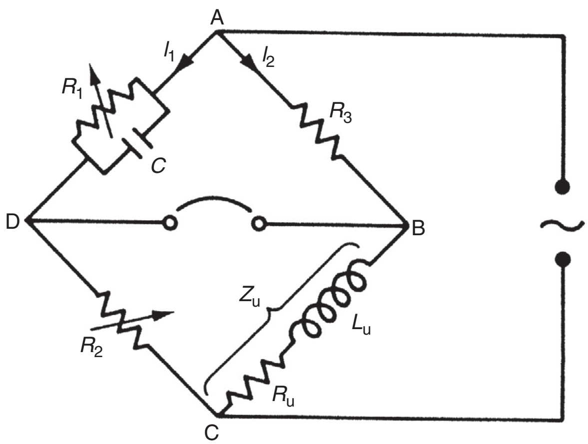

Figure 7.8
Maxwell bridge.

Thus,

$$\frac{Z_{BC}}{Z_{AB}} = \frac{Z_{DC}}{Z_{AD}} \quad \text{or} \quad Z_{BC} = \frac{Z_{DC}Z_{AB}}{Z_{AD}} \tag{7.12}$$

The quantities in Eqn (7.12) have the following values:

$$\frac{1}{Z_{AD}} = \frac{1}{R_1} + j\omega C \quad \text{or} \quad Z_{AD} = \frac{R_1}{1 + j\omega CR_1}$$

$$Z_{AB} = R_3 \ ; \ Z_{BC} = R_u + j\omega L_u \ ; \ Z_{DC} = R_2$$

Substituting the values into Eqn (7.12):

$$R_u + j\omega L_u = \frac{R_2R_3(1 + j\omega CR_1)}{R_1}$$

Taking real and imaginary parts:

$$R_u = \frac{R_2R_3}{R_1} \ ; \ L_u = R_2R_3C \tag{7.13}$$

This expression (7.13) can be used to calculate the quality factor (Q value) of the coil:

$$Q = \frac{\omega L_u}{R_u} = \frac{\omega R_2R_3CR_1}{R_2R_3} = \omega CR_1$$

If a constant frequency ω is used:

$$Q \approx R_1$$

Thus, the Maxwell bridge can be used to measure the Q value of a coil directly using this relationship.

Example 7.7

In the Maxwell bridge shown in Figure 7.8, let the fixed-value bridge components have the following values: $R_3 = 5\ \Omega$; $C = 1$ mF. Calculate the value of the unknown impedance (L_u, R_u), if $R_1 = 159\ \Omega$ and $R_2 = 10\ \Omega$ at balance.

Solution

Substituting values into the relations developed in Eqn (7.13) above:

$$R_u = \frac{R_2R_3}{R_1} = \frac{10 \times 5}{159} = 0.3145\Omega \quad ; \quad L_u = R_2R_3C = \frac{10 \times 5}{1000} = 50 \text{ mH}$$

Example 7.8

Calculate the Q factor for the unknown impedance in Example 7.7 above at a supply frequency of 50 Hz.

Solution

$$Q = \frac{\omega L_u}{R_u} = \frac{2\pi 50(0.05)}{0.3145} = 49.9$$

Deflection-type AC bridge

A common deflection type of AC bridge circuit is shown in Figure 7.9.

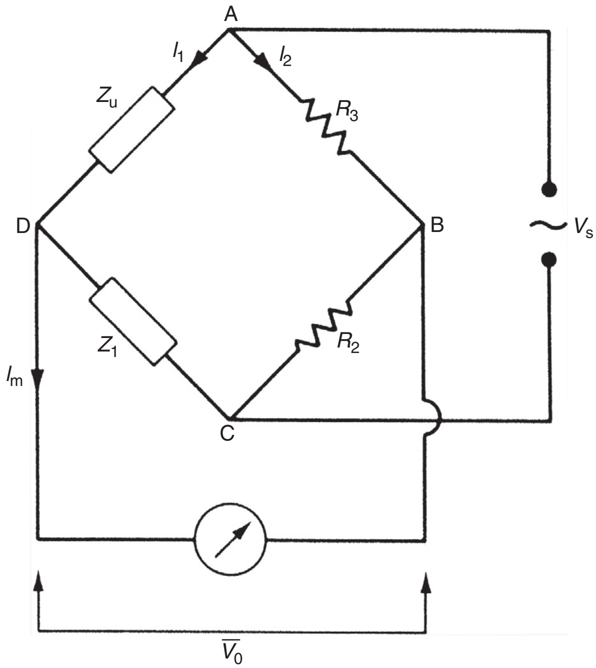

Figure 7.9
Common deflection-type AC bridge.

For capacitance measurement:

$$Z_u = 1/j\omega C_u \quad ; \quad Z_1 = 1/j\omega C_1$$

For inductance measurement (making the simplification that the resistive component of the inductor is small and approximates to zero):

$$Z_u = j\omega L_u \quad ; \quad Z_1 = j\omega L_1$$

Analysis of the circuit to find the relationship between V_0 and Z_u is greatly simplified if one assumes that I_m is negligible. This is valid, provided that the instrument measuring V_0 has a high impedance. For $I_m = 0$, currents in the two branches of the bridge, as defined in Figure 7.9, are given by:

$$I_1 = \frac{V_s}{Z_1 + Z_u} \quad ; \quad I_2 = \frac{V_s}{R_2 + R_3}$$

Also, $V_{AD} = I_1 Z_u$ and $V_{AB} = I_2 R_3$

Hence:

$$V_0 = V_{BD} = V_{AD} - V_{AB} = V_s\left(\frac{Z_u}{Z_1 + Z_u} - \frac{R_3}{R_2 + R_3}\right)$$

Thus, for capacitances:

$$V_0 = V_s\left(\frac{1/C_u}{1/C_1 + 1/C_u} - \frac{R_3}{R_2 + R_3}\right) = V_s\left(\frac{C_1}{C_1 + C_u} - \frac{R_3}{R_2 + R_3}\right) \tag{7.14}$$

and for inductances:

$$V_0 = V_s\left(\frac{L_u}{L_1 + L_u} - \frac{R_3}{R_2 + R_3}\right) \tag{7.15}$$

This latter relationship (7.15) in practice is only approximate since inductive impedances are never pure inductances as assumed but always contain a finite resistance (i.e., $Z_u = j\omega L_u + R$). However, the approximation is valid in many circumstances.

Example 7.9

A deflection bridge as shown in Figure 7.9 is used to measure an unknown capacitance, C_u. The components in the bridge have the following values:

$$V_s = 20V_{\text{rms}}, \quad C_1 = 100\ \mu\text{F}, \quad R_2 = 60\ \Omega, \quad R_3 = 40\ \Omega$$

If $C_u = 100\ \mu\text{F}$, calculate the output voltage V_0.

Solution

From Eqn (7.14):

$$V_0 = V_s\left(\frac{C_1}{C_1 + C_u} - \frac{R_3}{R_2 + R_3}\right) = 20(0.5 - 0.4) = 2V_{\text{rms}}$$

Example 7.10

An unknown inductance L_u is measured using a deflection type of bridge as shown in Figure 7.9. The components in the bridge have the following values:

$$V_s = 10V_{\text{rms}},\quad L_1 = 20\ \text{mH},\quad R_2 = 100\ \Omega,\quad R_3 = 100\ \Omega$$

If the output voltage V_0 is $1V_{\text{rms}}$, calculate the value of L_u.

Solution

From Eqn (7.15):

$$\frac{L_u}{L_1 + L_u} = \frac{V_0}{V_s} + \frac{R_3}{R_2 + R_3} = 0.1 + 0.5 = 0.6$$

Thus,

$$L_u = 0.6(L_1 + L_u)\quad ;\quad 0.4\ L_u = 0.6\ L_1\quad ;\quad L_u = \frac{0.6L_1}{0.4} = 30\ \text{mH}$$

7.2.5 Commercial Bridges

Ready-built bridges are available commercially, although these are substantially more expensive than a “home-made” bridge made up from discrete components and a voltmeter to measure the output voltage.

7.3 Resistance Measurement

Devices that convert the measured quantity into a change in resistance include the resistance thermometer, the thermistor, the wire-coil pressure gauge, and the strain gauge.

The standard devices and methods available for measuring change in resistance, which is measured in units of *ohms* (Ω), include the DC bridge circuit, the voltmeter–ammeter method, the resistance-substitution method, the digital voltmeter, and the ohmmeter. Apart from the ohmmeter, these instruments are normally used to measure medium values of resistance in the range of 1 Ω to 1 MΩ, but this range is entirely adequate for all current sensors that convert the measured quantity into a change in resistance.

7.3.1 DC Bridge Circuit

DC bridge circuits, as discussed earlier, provide the most commonly used method of measuring medium resistance values. The best measurement accuracy is provided by the null-output-type Wheatstone bridge, and inaccuracy values of less than ±0.02% are achievable with commercially available instruments. Deflection-type bridge circuits are simpler to use in practice than the null-output-type, but their measurement accuracy is inferior and the nonlinear output relationship is an additional difficulty. Bridge circuits are particularly useful in converting resistance changes into voltage signals that can be input directly into automatic control systems.

7.3.2 Voltmeter–Ammeter Method

The voltmeter–ammeter method consists of applying a measured DC voltage across the unknown resistance and measuring the current flowing. Two alternatives exist for connecting the two meters, as shown in Figure 7.10. In Figure 7.10(a), the ammeter measures the current flowing in both the voltmeter and the resistance. The error due to this is minimized when the measured resistance is small relative to the voltmeter resistance. In the alternative form of connection, Figure 7.10(b), the voltmeter measures the voltage drop across the unknown resistance and the ammeter. Here, the measurement error is minimized when the unknown resistance is large with respect to the ammeter resistance. Thus, method (a) is best for the measurement of small resistances and method (b) for large ones.

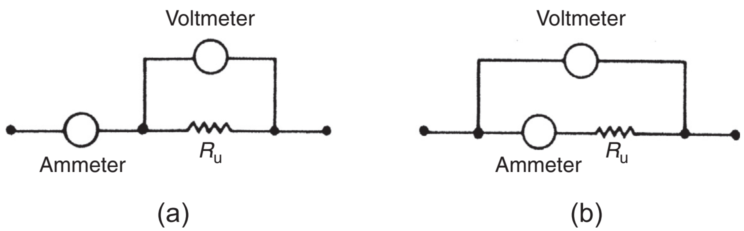

Figure 7.10
Voltmeter–ammeter method of measuring resistance.

Having thus measured the voltage and current, the value of the resistance is then calculated very simply by Ohm's law. This is a suitable method wherever the measurement inaccuracy of up to ±1% that it gives is acceptable.

7.3.3 Resistance-Substitution Method

In the voltmeter—ammeter method above, either the voltmeter is measuring the voltage across the ammeter as well as across the resistance, or the ammeter is measuring the current flow through the voltmeter as well as through the resistance. The measurement error caused by this is avoided in the resistance-substitution technique. In this method, the unknown resistance in a circuit is temporarily replaced by a variable resistance. The variable resistance is adjusted until the measured circuit voltage and current are the same as existed with the unknown resistance in place. The variable resistance at this point is equal in value to the unknown resistance.

7.3.4 Use of the Digital Voltmeter to Measure Resistance

The digital voltmeter can also be used for measuring resistance if an accurate current source is included within it that passes current through the resistance. This can give a measurement inaccuracy as small as ±0.1%.

7.3.5 The Ohmmeter

Ohmmeters are used to measure resistances over a wide range from a few milliohms up to 50 MΩ. The first generation of ohmmeters contained a battery that applied a known voltage across a combination of the unknown resistance and a known resistance in series, as shown in Figure 7.11. Measurement of the voltage, V_m, across the known resistance, R, allows the unknown resistance, R_u, to be calculated from:

$$R_u = \frac{R(V_b - V_m)}{V_m}$$

where V_b is the battery voltage. Unfortunately, this mode of resistance measurement gives a typical inaccuracy of ±2%, which is only acceptable in a very limited number of applications. Because of this, first-generation ohmmeters have been mostly replaced by a new type of electronic ohmmeter.

The electronic ohmmeter contains two circuits. The first circuit generates a constant current (I) that is passed through the unknown resistance. The second circuit measures the voltage (V) across the resistance. The resistance is then given by Ohm's law as: $R = V/I$. Electronic ohmmeters can achieve measurement inaccuracy as low as ±0.02%.

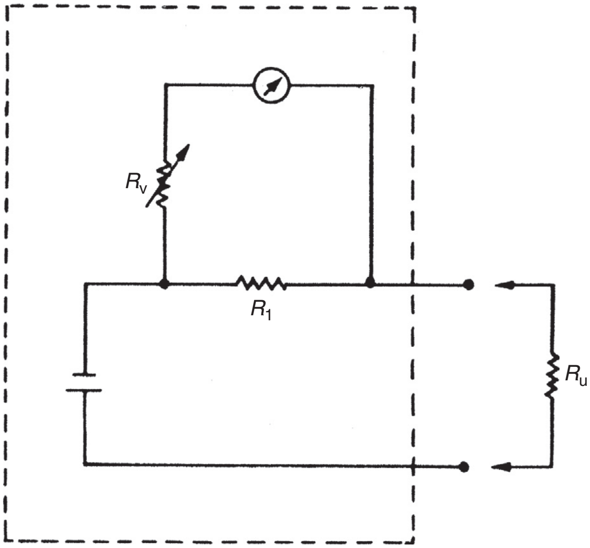

Figure 7.11
Ohmmeter.

Most *digital and analog multimeters* contain circuitry of the same form as in an ohmmeter, and hence can be similarly used to obtain measurements of resistance.

7.4 Inductance Measurement

The main device that has an output in the form of a change in inductance is the inductive displacement sensor. Inductance is measured in *Henry* (H). It can only be measured accurately by an AC bridge circuit, and various commercial inductance bridges are available. However, when such a commercial inductance bridge is not immediately available, the following method can be applied to give an approximate measurement of inductance.

This approximate method consists of connecting the unknown inductance in series with a variable resistance, in a circuit excited with a sinusoidal voltage, as shown in Figure 7.12. The variable resistance is adjusted until the voltage measured across the resistance is equal to that measured across the inductance. The two impedances are then equal, and the value of the inductance L can be calculated from:

$$L = \frac{\sqrt{(R^2 - r^2)}}{2\pi f}$$

where R is the value of the variable resistance, r is the value of the inductor resistance, and f is the excitation frequency.

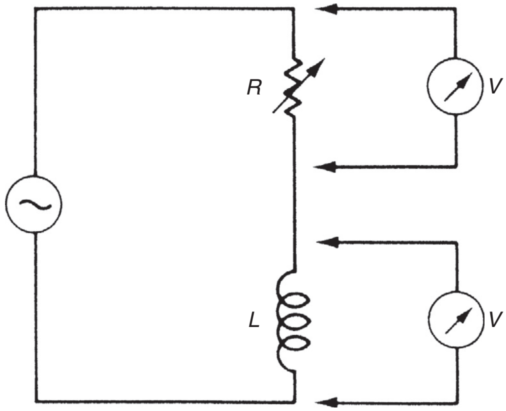

Figure 7.12
Approximate method of measuring inductance.

7.5 Capacitance Measurement

Devices that have an output in the form of a change in capacitance include the capacitive level gauge, the capacitive displacement sensor, the capacitive moisture meter, and the capacitive hygrometer. Capacitance is measured in units of *Farads* (F). Like inductance, capacitance can only be measured accurately by an AC bridge circuit, and various types of capacitance bridges are available commercially. In circumstances where a proper capacitance bridge is not immediately available, and if an approximate measurement of capacitance is acceptable, one of the following two methods can be considered.

The first of these, shown in Figure 7.13, consists of connecting the unknown capacitor in series with a known resistance in a circuit excited at a known frequency. An AC voltmeter is used to measure the voltage drop across both the resistor and the capacitor. The capacitance value is then given by:

$$C = \frac{V_r}{2\pi f R V_c}$$

where V_r and V_c are the voltages measured across the resistance and capacitance, respectively, f is the excitation frequency, and R is the known resistance.

An alternative approximate method of measurement is to measure the time constant of the capacitor connected in an *RC* circuit.

7.6 Current Measurement

Current measurement is needed for devices like the thermocouple-gauge pressure sensor and the ionization gauge that have an output in the form of a varying electrical current. It

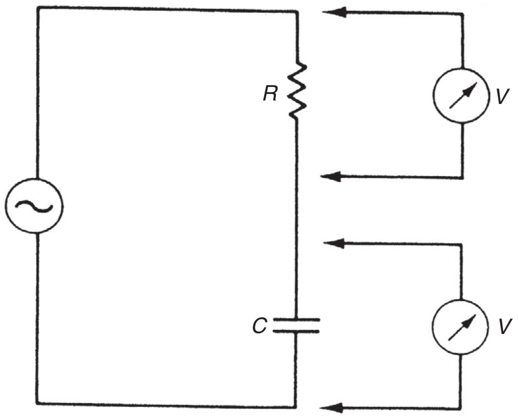

Figure 7.13
Approximate method of measuring capacitance.

is often also needed in signal transmission systems that convert the measured signal into a varying current. Any of the digital and analog voltmeters discussed later in Chapter 9 can measure current if the meter is placed in series with the current-carrying circuit, and the same frequency limits apply for the measured signal as they do for voltage measurement. The upper frequency limit for AC current measurement can be raised by rectifying the current prior to measurement. To minimize the loading effect on the measured system, any current-measuring instrument must have a small resistance. This is opposite to the case of voltage measurement where the instrument is required to have a high resistance for minimal circuit loading.

Besides the requirement to measure signal-level currents, many measurement applications also require higher magnitude electrical currents to be measured. Hence, the following discussion covers the measurement of currents at both signal level and higher magnitudes.

Analog meters are useful in applications where there is a need to display the measured value on a control panel. Moving coil instruments are used as panel meters to measure DC current in the milliamp range up to 1 A. Moving iron meters can measure both DC and AC up to several hundred amps directly. To measure larger currents with electromechanical meters, it is necessary to insert a shunt resistance into the circuit and measure the voltage drop across it. Apart from the obvious disturbance of the measured system, one particular difficulty that results from this technique is the large power dissipation in the shunt. In the case of AC current measurement, care must also be taken to match the resistance and reactance of the shunt to that of the measuring instrument so that frequency and waveform distortion in the measured signal are avoided.

Current transformers provide an alternative method of measuring high-magnitude currents that avoids the difficulty of designing a suitable shunt. Different versions of these exist for

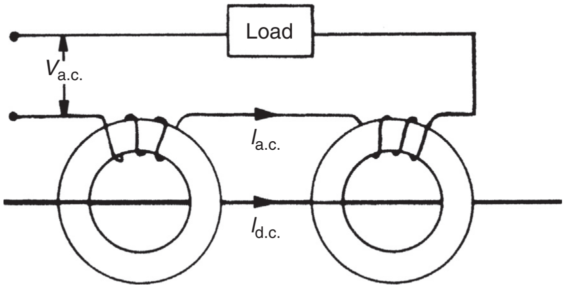

Figure 7.14
Current transformer.

transforming both DC and AC currents. A DC current transformer is shown in Figure 7.14. The central DC conductor in the instrument is threaded through two magnetic cores that carry two high-impedance windings connected in series opposition. It can be shown that the current flowing in the windings when excited with an AC voltage is proportional to the DC current in the central conductor. This output current is commonly rectified and then measured by a DC voltmeter.

An AC transformer typically has a primary winding consisting of only a few copper turns wound on a rectangular or ring-shaped core. The secondary winding, on the other hand, would normally have several hundred turns according to the current step-down ratio required. The output of the secondary winding is measured by any suitable current-measuring instrument. The design of current transformers is substantially different from that of voltage transformers. The rigidity of its mechanical construction has to be sufficient to withstand the large forces arising from short-circuit currents, and special attention has to be paid to the insulation between its windings for similar reasons. A low-loss core material is used and flux densities are kept as small as possible to reduce losses. In the case of very high currents, the primary winding often consists of a single copper bar that behaves as a single-turn winding. The clamp-on meter, described later in Chapter 9, is a good example of this.

All of the other instruments for measuring the voltage discussed in Chapter 9 can be applied to current measurement by using them to measure the voltage drop across a known resistance placed in series with the current-carrying circuit. The digital voltmeter is widely applied for measuring currents accurately by this method, and the oscilloscope is frequently used to obtain approximate measurements in circuit-test applications. Finally, mention must also be made of the use of digital and analog multimeters for current measurement, particularly in circuit-test applications. These instruments include a set of switchable dropping resistors and so can measure currents over a wide range. Protective circuitry within such instruments prevents damage when high currents are applied on the wrong input range.

7.7 Frequency Measurement

Frequency measurement is required as part of those devices that convert the measured physical quantity into a frequency change, such as the variable-reluctance velocity transducer, stroboscopes, the vibrating-wire force sensor, resonant-wire pressure sensor, the turbine flowmeter, the Doppler shift ultrasonic flowmeter, the transit time ultrasonic flowmeter, the vibrating-level sensor, the quartz moisture meter, and the quartz thermometer. In addition, the output relationship in some forms of the AC bridge circuit used for measuring inductance and capacitance requires accurate measurement of the bridge excitation frequency.

Frequency is measured in units of *Hertz* (Hz). The digital counter-timer is the most common instrument for measuring frequency. Alternatively, a phase-locked loop can be used. The oscilloscope is also commonly used, especially in circuit-test and fault-diagnosis applications. Finally, for measurements within the audio frequency range, the Wien bridge is a further instrument that is sometimes used.

7.7.1 Digital Counter-Timers

A digital counter-timer is the most accurate and flexible instrument available for measuring frequency. Inaccuracy can be reduced down to one part in 10^8, and all frequencies between DC and several gigahertz can be measured. The essential component within a counter-timer instrument is an oscillator that provides a very accurately known and stable reference frequency, which is typically either 100 kHz or 1 MHz. This is often maintained in a temperature-regulated environment within the instrument to guarantee its accuracy. The oscillator output is transformed by a pulse shaper circuit into a train of pulses and applied to an electronic gate, as shown in Figure 7.15. Successive pulses at the reference frequency alternately open and close the gate. The input signal of the unknown frequency is similarly transformed into a train of

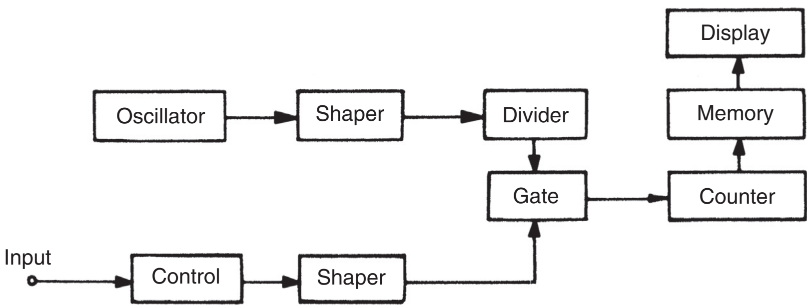

Figure 7.15
Digital counter-timer system.

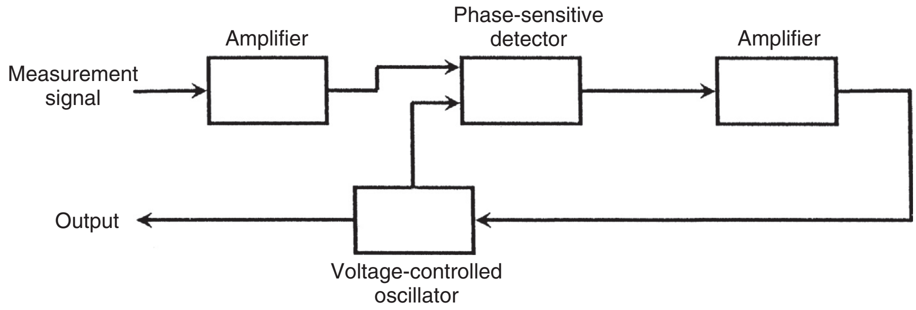

Figure 7.16
Phase-locked loop.

pulses and applied to the gate. The number of these pulses that get through the gate during the time that it is open during each gate cycle is proportional to the frequency of the unknown signal.

The accuracy of measurement obviously depends upon how far the unknown frequency is above the reference frequency. As it stands therefore, the instrument can only accurately measure frequencies that are substantially above 1 MHz. To enable the instrument to measure much lower frequencies, a series of decade frequency dividers are provided within it. These increase the time between the reference frequency pulses by factors of 10, and a typical instrument can have gate pulses separated in time between 1 μs and 1 s.

Improvement in the accuracy of low-frequency measurement can be obtained by modifying the gating arrangements such that the signal of unknown frequency is made to control the opening and closing of the gate. The number of pulses at the reference frequency that pass through the gate during the open period is then a measure of the frequency of the unknown signal.

7.7.2 Phase-Locked Loop

A phase-locked loop is a circuit consisting of a phase-sensitive detector, a voltage-controlled oscillator (VCO), and amplifiers, connected in a closed-loop system as shown in Figure 7.16. In a VCO, the oscillation frequency is proportional to the applied voltage. Operation of a phase-locked loop is as follows. The phase-sensitive detector compares the phase of the amplified input signal with the phase of the VCO output. Any phase difference generates an error signal, which is amplified and fed back to the VCO. This adjusts the frequency of the VCO until the error signal goes to zero, and thus the VCO becomes locked to the frequency of the input signal. The DC output from the VCO is then proportional to the input signal frequency.

7.7.3 Oscilloscope

Many digital oscilloscopes (particularly the more expensive ones) have a push button on the front panel that causes the instrument to automatically compute and display the frequency of the input signal as a numeric value.

Where this direct facility is not available (in some digital oscilloscopes and all analog ones), two alternative ways of using the instrument to measure frequency are available. First, the internal time base can be adjusted until the distance between two successive cycles of the measured signal can be read against the calibrated graticule on the screen. Measurement accuracy by this method is limited, but can be optimized by measuring between points in the cycle where the slope of the waveform is steep, generally where it is crossing through from the negative to the positive part of the cycle. Calculation of the unknown frequency from this measured time interval is relatively simple. For example, suppose that the distance between the two cycles is 2.5 divisions when the internal time base is set at 10 ms/division. The cycle time is therefore 25 ms and hence the frequency is 1000/25, that is, 40 Hz. Measurement accuracy is dependent upon how accurately the distance between the two cycles is read, and it is very difficult to reduce the error level below $\pm 5\%$ of the reading.

The alternative way of using an oscilloscope to measure frequency is to generate *Lissajous patterns*. These are produced by applying a known reference-frequency sine wave to the y input (vertical deflection plates) of the oscilloscope and the unknown frequency sinusoidal signal to the x input (horizontal deflection plates). A pattern is produced on the screen according to the frequency ratio between the two signals, and if the numerator and denominator in the ratio of the two signals both represent an integral number of cycles, the pattern is stationary. Examples of these patterns are shown in Figure 7.17, which also shows that the phase difference between the waveforms has an effect on the shape. Frequency measurement proceeds by adjusting the reference frequency until a steady pattern is obtained on the screen and then calculating the unknown frequency according to the frequency ratio that the pattern obtained represents.

7.7.4 The Wien Bridge

The Wien bridge, shown in Figure 7.18, is a special form of AC bridge circuit that can be used to measure frequencies in the audio range. An alternative use of the instrument is as a source of audio frequency signals of accurately known frequency. A simple set of headphones is often used to detect the null-output balance condition. Other suitable instruments for this purpose are the oscilloscope and the electronic voltmeter. At balance, the unknown frequency is calculated according to:

$$f = \frac{1}{2\pi R_3 C_3}$$

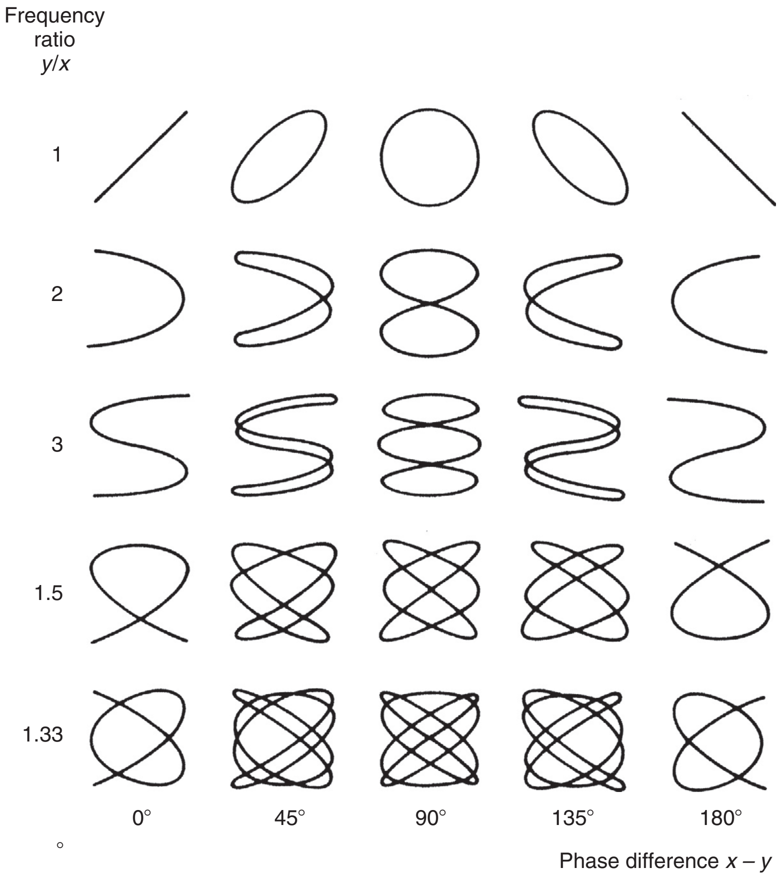

Figure 7.17
Lissajous patterns.

The instrument is very accurate at audio frequencies, but at higher frequencies errors due to losses in the capacitors and stray capacitance effects become significant.

7.8 Phase Measurement

Instruments that convert the measured variable into a phase change in a sinusoidal electrical signal include the transit time ultrasonic flowmeter, the radar-level sensor, the linear variable differential transformer (LVDT), and the resolver. The most accurate instrument for measuring the phase difference between two signals is the electronic counter-timer. However, other methods also exist. These include plotting the signals on an $X-Y$ plotter, using an oscilloscope, and using a phase-sensitive detector.

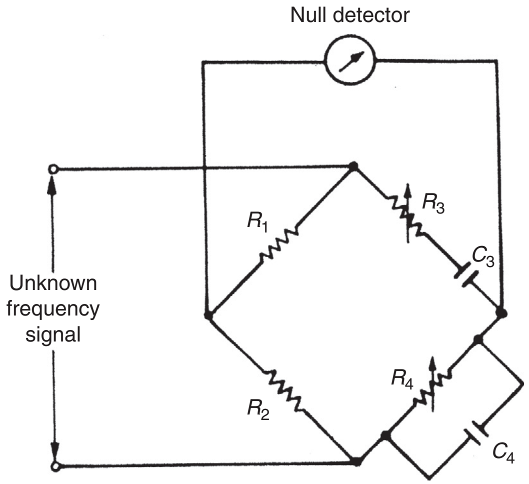

Figure 7.18
Wien bridge.

7.8.1 Electronic Counter-Timer

In principle, the phase difference between the two sinusoidal signals can be determined by measuring the time that elapses between the two signals crossing the time axis. However, in practice, this is inaccurate because the zero crossings are susceptible to noise contamination. The normal solution to this problem is to amplify/attenuate the two signals so that they have the same amplitude and then measure the time that elapses between the two signals crossing some nonzero threshold value.

The basis of this method of phase measurement is a digital counter-timer with a quartz-controlled oscillator providing a frequency standard that is typically 10 MHz. The crossing points of the two signals through the reference threshold voltage level are applied to a gate that starts and then stops pulses from the oscillator into an electronic counter, as shown in Figure 7.19. The elapsed time, and hence phase difference, between the two input signals is then measured in terms of the counter display.

7.8.2 X—Y Plotter

This is a useful technique for approximate phase measurement but is limited to low frequencies because of the very limited bandwidth of an X—Y plotter. If two input signals of equal magnitude are applied to the X and Y inputs of a plotter, the plot obtained is an ellipse, as shown in Figure 7.20. The X and Y inputs are given by:

$$V_X = V\sin(\omega t) \quad ; \quad V_Y = V\sin(\omega t + \phi)$$

At $t = 0$, $V_X = 0$, and $V_Y = V\sin\phi$. Thus, from Figure 7.20, for $V_X = 0$, $V_Y = \pm h$:

$$\sin\phi = \pm h/V \tag{7.16}$$

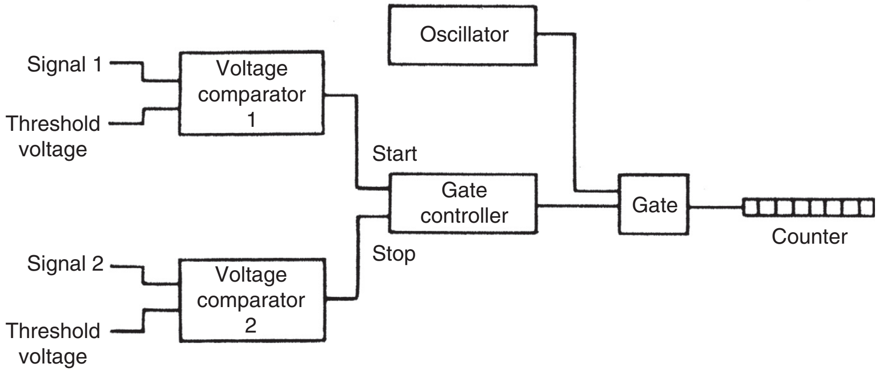

Figure 7.19
Phase measurement with digital counter-timer.

Solution of Eqn (7.4) gives four possible values for ϕ, but the ambiguity about which quadrant ϕ is in can usually be solved by observing the two signals plotted against time on a dual-beam oscilloscope.

7.8.3 Oscilloscope

As for the case of frequency measurement, many digital oscilloscopes (particularly the more expensive ones) have a push button on the front panel that causes the instrument to automatically compute and display the phase of the input signal as a numeric value.

Where this direct facility is not available (in some digital oscilloscopes and all analog ones), approximate measurement of the phase difference between signals can be made using any dual-beam oscilloscope. The two signals are applied to the two oscilloscope

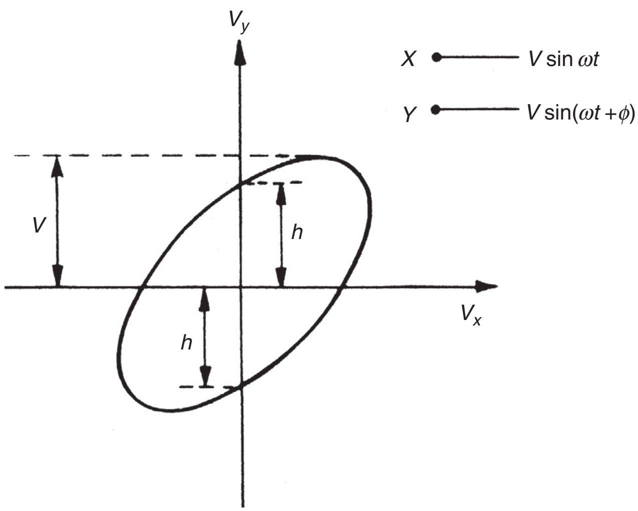

Figure 7.20
Phase measurement using an X–Y plotter.

inputs and a suitable time base chosen such that the time between the crossing points of the two signals can be measured. The phase difference of both low- and high-frequency signals can be measured by this method, the upper frequency limit measurable being dictated by the bandwidth of the oscilloscope (which is normally very high).

7.8.4 Phase-Sensitive Detector

A phase-sensitive detector can be used to measure the phase difference between two signals that have an identical frequency. Phase-sensitive detectors are also known by several alternative names, two examples of which are synchronous demodulator and synchronous detector. They can also exist physically in a number of alternative forms that include both transformer-based and fully electronic circuits. For two signals of amplitude V_1 and V_2, with the same frequency f, the output is given by $V_1V_2cos\phi$, where ϕ is the phase difference between the signals. This can be exploited in measurement devices like the varying-phase output resolver (see Chapter 20).

7.9 Summary

This chapter has been concerned with looking at ways of dealing with outputs from a measurement sensor that are not in the form of a readily measureable voltage signal. We started off by identifying the various alternative forms of output that we might have to deal with. Our list included translational displacement change outputs; changes in various electrical parameters such as resistance, inductance, capacitance, and current; and changes in the phase or frequency of an AC electrical signal.

This led us to realize that we needed mechanisms for converting these sensor outputs that are initially in some nonvoltage form into a more convenient form. Such mechanisms are collectively called variable conversion elements. Since mechanisms for measuring translational displacements are needed for other purposes as well, we have deferred consideration of these until Chapter 19, where the subject of translational measurement is considered in detail. The rest of this chapter has therefore only been concerned with looking at the ways of dealing with nonvoltage electrical parameter outputs and outputs in the form of a frequency of phase change in an electrical signal.

We learned first of all in this study that bridge circuits are a particularly important type of variable conversion element, and we therefore went on to cover these in some detail. One particularly important thing that we learned was that bridge circuits existed in two forms, null type and deflection type. Of these, null types are more tedious to use but provide better measurement accuracy, leading to these being the preferred form when sensors are being calibrated. We noted also that both DC and AC bridges exist, the former being used to interpret the output of sensors that exhibit a change in resistance and the latter for sensors that convert the measured quantity into a change in either inductance or capacitance.

We then went on to look at the ways of dealing with sensor outputs in other forms. In turn, we covered resistance measurement (alternative ways of using a DC bridge circuit), inductance measurement, capacitance measurement, and current measurement. Finally, we looked at ways of interpreting the output of sensors that is in the form of a change in either the frequency or the phase of an electrical signal.

7.10 Problems

7.1 A null-type Wheatstone bridge is used to accurately measure the resistance of a nickel resistance thermometer during a calibration procedure. The circuit shown in Figure 7.1 is used, in which the known fixed resistance values are given by $R_2 = 123.7\ \Omega$ and $R_3 = 127.4\ \Omega$. The thermometer is inserted in the circuit as R_u and then the variable resistance box R_v is adjusted until the bridge output voltage V_0 goes to zero. At this balance point, the value of R_v is 117.3 Ω. Calculate the resistance of the thermometer.

7.2 Explain what a DC bridge circuit is and why it is so useful in measurement systems. List a few measurement sensors for which you would commonly use a DC bridge circuit to convert the sensor output into a change in output voltage of the bridge.

7.3 If elements in the DC bridge circuit shown in Figure 7.2 have the following values: $R_u = 110\ \Omega$, $R_1 = 100\ \Omega$, $R_2 = 1000\ \Omega$, $R_3 = 1000\ \Omega$, $V_i = 10$ V, calculate the output voltage V_0, if the impedance of the voltage-measuring instrument is assumed to be infinite. (Hint: Apply Eqn (7.3).)

7.4 A null-type Wheatstone bridge is used to accurately measure the resistance of a strain gauge during a calibration procedure. The circuit shown in Figure 7.1 is used, in which the known fixed resistance values are given by $R_2 = 331.2\ \Omega$ and $R_3 = 327.5\ \Omega$. The thermometer is inserted in the circuit as R_u and then the variable resistance box R_v is adjusted until the bridge output voltage V_0 goes to zero. At this balance point, the value of R_v is 352.5 Ω. Calculate the resistance of the strain gauge.

7.5 Suppose that the resistive components in the DC bridge shown in Figure 7.2 have the following nominal values: $R_u = 3$ kΩ; $R_1 = 6$ kΩ; $R_2 = 8$ kΩ; $R_3 = 4$ kΩ. The actual value of each resistance is related to the nominal value according to $R_{actual} = R_{nominal} + \partial R$, where ∂R has the following values: $\partial R_u = 30\ \Omega$; $\partial R_1 = -20\ \Omega$; $\partial R_2 = 40\ \Omega$; $\partial R_3 = -50\ \Omega$. Calculate the open circuit bridge output voltage if the bridge supply voltage V_i is 50 V.

7.6 (a) Suppose that the unknown resistance R_u in Figure 7.2 is a resistance thermometer whose resistance at 100 °C is 500 Ω and whose resistance varies with temperature at the rate of 0.5 Ω/°C for small-temperature changes around 100 °C. Calculate the sensitivity of the total measurement system for small changes in temperature around 100 °C, given the following resistance and voltage values measured at 15 °C by instruments calibrated at 15 °C: $R_1 = 500\ \Omega$; $R_2 = R_3 = 5000\ \Omega$; $V_i = 10$ V.

(b) If the resistance thermometer is measuring a fluid whose true temperature is 104 °C, calculate the error in the indicated temperature if the ambient temperature around the bridge circuit is 20 °C instead of the calibration temperature of 15 °C, given the following additional information:
Voltage-measuring instrument zero drift coefficient = +1.3 mV/°C,
Voltage-measuring instrument sensitivity drift coefficient = 0,
Resistances R_1, R_2, and R_3 have a positive temperature coefficient of +0.2% of nominal value/°C,
Voltage source V_i is unaffected by temperature changes.

7.7 Suppose that the resistive components in the DC bridge shown in Figure 7.2 have the following nominal values: $R_u = 330\ \Omega$; $R_1 = 470\ \Omega$; $R_2 = 560\ \mathrm{k}\Omega$; $R_3 = 270\ \mathrm{k}\Omega$. The actual value of each resistance is related to the nominal value according to: $R_{\mathrm{actual}} = R_{\mathrm{nominal}} + \partial R$ where ∂R has the following values: $\partial R_u = -5\ \Omega$; $\partial R_1 = +4\ \Omega$; $\partial R_2 = +6\ \Omega$; $\partial R_3 = -9\ \Omega$. Calculate the open circuit bridge output voltage if the bridge supply voltage V_i is 12 V.

7.8 Four strain gauges of resistance 120 Ω each are arranged into a DC bridge configuration such that each of the four arms in the bridge has one strain gauge in it. The maximum permissible current in each strain gauge is 100 mA. What is the maximum bridge supply voltage allowable, and what power is dissipated in each strain gauge with that supply voltage?

7.9 (a) Suppose that the variables shown in Figure 7.2 have the following values: $R_1 = 100\ \Omega$, $R_2 = 100\ \Omega$, $R_3 = 100\ \Omega$; $V_i = 12$ V. R_u is a resistance thermometer with a resistance of 100 Ω at 100 °C and a temperature coefficient of +0.3 Ω/°C over the temperature range from 50 to 150 °C (i.e., the resistance increases as the temperature goes up). Draw a graph of bridge output voltage V_0 for ten-degree steps at a temperature between 100 and 150 °C (calculating V_0 according to Eqn (7.3)).

(b) Briefly discuss whether you expect the graph that you have just drawn to be a straight line.

(c) Draw a graph of V_0 for similar temperature values if $R_2 = R_3 = 1000\ \Omega$ and all other components have the same values as given in part (a) above. Note that the line through the data points is straighter than that drawn in part (a) but the output voltage is much less at each temperature point.

(d) Briefly discuss the change in linearity of the graph drawn for part (c) and the change in measurement sensitivity compared with the graph drawn for part (a).

7.10 Four strain gauges of resistance 350 Ω each are arranged into a DC bridge configuration such that each of the four arms in the bridge has one strain gauge in it. The maximum permissible current in each strain gauge is 30 mA. What is the maximum bridge supply voltage allowable, and what power is dissipated in each strain gauge with that supply voltage?

7.11 The unknown resistance R_u in a dc bridge circuit, connected as shown in Figure 7.4(a), is a resistance thermometer. The thermometer has a resistance of 350 Ω at 50 °C and its temperature coefficient is +1 Ω/°C (the resistance increases as the temperature rises). The components of the system have the following values: $R_1 = 350\ \Omega$, $R_2 = R_3 = 2\ \mathrm{k}\Omega$, $R_m = 20\ \mathrm{k}\Omega$, $V_i = 5$ V. What is the output voltage reading when the temperature is 100 °C? (Hint: use Eqn (7.10).)

7.12 The active element in a load cell is a strain gauge with a nominal resistance of 500 Ω in its unstressed state. The cell has a sensitivity of +0.5 Ω per Newton of applied force and is connected in a DC bridge circuit where the other three arms of the bridge each have a resistance of 500 Ω.

(a) If the bridge excitation voltage is 20 V, what is the measurement sensitivity of the system in volts/Newton for small applied forces?

(b) What is the bridge output voltage when measuring an applied force of 500 N?

7.13 Suppose that the unknown resistance R_u in Figure 7.2 is a resistance thermometer whose resistance at 100 °C is 600 Ω and whose resistance varies with temperature at the rate of +0.4 Ω/°C for small temperature changes around 100 °C. Calculate the sensitivity of the total measurement system for small changes in temperature around 100 °C, given the following resistance and voltage values:

$$R_1 = 600\ \Omega \quad ; \quad R_2 = R_3 = 6000\ \Omega \quad ; \quad V_i = 20\ \mathrm{V}$$

Assume that the ambient temperature around the bridge circuit was the same as that at which the voltage-measuring instrument and all bridge component values were calibrated.

7.14 The unknown resistance R_u of a resistance thermometer is measured by a deflection-type bridge circuit of the form shown in Figure 7.2, where the parameters have the following values:

$$R_1 = 100\ \Omega \quad ; \quad R_2 = R_3 = 1000\ \Omega \quad ; \quad V_i = 20\ \mathrm{V}$$

The thermometer has a resistance of 100 Ω at 0 °C and the resistance varies with temperature at the rate of 0.4 Ω/°C for small temperature changes around 0 °C.

(a) Calculate the bridge sensitivity in units of volts/ohm.

(b) Calculate the sensitivity of the total measurement system in units of volts/°C for small-temperature changes around 0 °C.

7.15 The unknown resistance R_u of a resistance thermometer is to be measured by a bridge circuit of the form shown in Figure 7.5 where the bridge components and the excitation voltage are different to the values shown in Figure 7.5, having instead the following values:
Nominal thermometer resistance at temperature of 20 °C = 100 Ω; $R_m = 10\ \mathrm{k}\Omega$ (unchanged);

$$R_1 = 100\ \Omega, \quad R_2 = 1000\ \Omega, \quad R_3 = 1000\ \Omega, \quad V_i = 10\ \mathrm{V}$$

(a) Using Thevenin's theorem, derive an expression for the sensitivity of the bridge in terms of the change in output voltage V_m that occurs when there is a small change in the resistance of the thermometer.

(b) If the resistance thermometer has a sensitivity of 400 mΩ/°C, calculate the temperature measurement sensitivity in bridge output volts (V_m) per °C.

7.16 The unknown resistance R_u of a thermistor is to be measured by a bridge circuit of the form shown in Figure 7.5 where the bridge components and the excitation voltage are different to the values shown in Figure 7.5, having instead the following values:

$$R_1 = 1000\ \Omega, \quad R_2 = 1000\ \Omega, \quad R_3 = 1000\ \Omega, \quad V_i = 10\ \text{V}, \quad R_m = 20\ \text{k}\Omega$$

The resistance (R_u) of the thermistor is related to the measured temperature (T) in degrees Kelvin (°K) according to the following expression:

$$R_u = 1000 \exp\left[3675\left(\frac{1}{T} - 0.003354\right)\right]$$

Draw a graph of the bridge output in volts for values of the measured temperature in steps of 5 °C between 0 and 50 °C.

7.17 In the DC bridge circuit shown in Figure 7.21, the resistive components have the following values:

$$R_1 = R_2 = 120\ \Omega; \quad R_3 = 117\ \Omega; \quad R_4 = 123\ \Omega; \quad R_a = R_p = 1000\ \Omega.$$

(a) What are the resistance values of the parts of the potentiometer track either side of the slider when the potentiometer is adjusted to balance the bridge?

(b) What then is the effective resistance of each of the two left-hand arms of the bridge when the bridge is balanced?

(*Note*: This question will involve the solution of a quadratic equation. For a quadratic equation of the form: $ax^2 + bx + c = 0$, the solution is given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.)

7.18 The unknown resistance R_u of a strain gauge is to be measured by a bridge circuit of the form shown in Figure 7.5 where the bridge components and the excitation voltage are different to the values shown in Figure 7.5, having instead the following values: Nominal strain gauge resistance = 350 Ω; $R_m = 8$ kΩ; $R_1 = 350$ Ω; $R_2 = 2000$ Ω; $R_3 = 2000$ Ω; $V_i = 24$ V.

(a) Using Thevenin's theorem, derive an expression for the sensitivity of the bridge in terms of the change in output voltage V_m that occurs when there is a small change in the resistance of the strain gauge.

(b) If the strain gauge is part of a pressure transducer with a sensitivity of 1.2 Ω/bar, calculate the pressure measurement sensitivity in bridge output volts (V_m) per bar.

7.19 In the DC bridge circuit shown in Figure 7.21, the resistive components have the following values: $R_1 = R_2 = 350$ Ω; $R_3 = 341$ Ω; $R_4 = 359$ Ω; $R_a = R_p = 3000$ Ω.

(a) What are the resistance values of the parts of the potentiometer track on either side of the slider when the potentiometer is adjusted to balance the bridge?

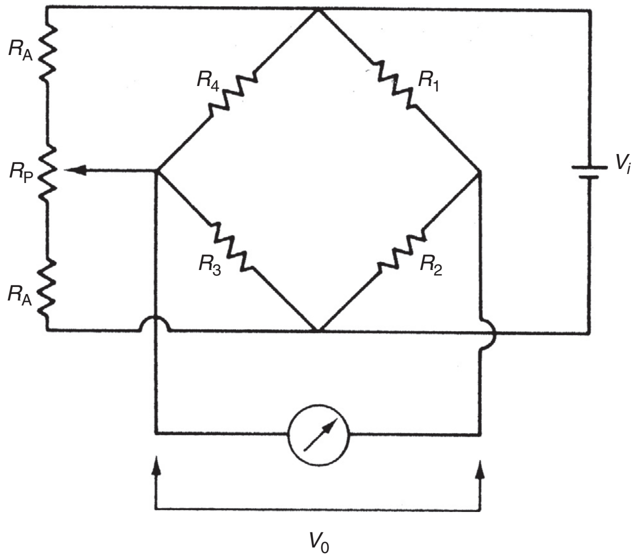

Figure 7.21
DC bridge with apex balancing.

(b) What then is the effective resistance of each of the two left-hand arms of the bridge when the bridge is balanced?

(*Note*: This question will involve the solution of a quadratic equation. For a quadratic equation of the form: $ax^2 + bx + c = 0$, the solution is given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.)

7.20 List a few measurement transducers and sensors for which you would commonly use an AC bridge circuit to convert the sensor output into a change in output voltage of the bridge.

7.21 A Maxwell bridge, designed to measure the unknown impedance (R_u, L_u) of a coil, is shown in Figure 7.8.

(a) Derive an expression for R_u and L_u under balance conditions.

(b) If the fixed bridge component values are $R_3 = 100\ \Omega$ and $C = 20\ \mu\text{F}$, calculate the value of the unknown impedance if $R_1 = 3183\ \Omega$ and $R_2 = 50\ \Omega$ at balance.

(c) Calculate the Q factor for the coil if the supply frequency is 50 Hz.

7.22 The deflection-type AC bridge shown in Figure 7.9 is used to measure an unknown inductance L_u. The components in the bridge have the following values: $V_s = 30V_{\text{rms}}$, $L_1 = 80$ mH, $R_2 = 70\ \Omega$, $R_3 = 30\ \Omega$. If $L_u = 50$ mH, calculate the output voltage V_0.

7.23 An unknown capacitance C_u is measured using a deflection bridge as shown in Figure 7.9. The components of the bridge have the following values: $V_s = 10V_{\text{rms}}$, $C_1 = 50\ \mu\text{F}$, $R_2 = 80\ \Omega$, $R_3 = 20\ \Omega$. If the output voltage is $3V_{\text{rms}}$, calculate the value of C_u.

7.24 A Hays bridge is often used for measuring the inductance of high Q coils and has the configuration shown in Figure 7.22. The inductance and resistance of the coil are represented in the figure by the symbols L_1 and R_1, respectively.

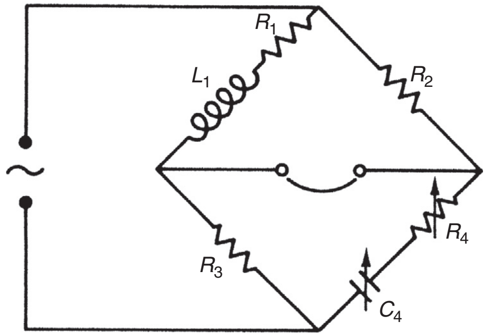

Figure 7.22
Hays bridge.

(a) Obtain the bridge balance conditions.
(b) Show that if the Q value of an unknown inductor coil is high, the expression for the inductance value when the bridge is balanced is independent of frequency.
(c) If the Q value is high, calculate the value of the inductor if the bridge component values at balance are as follows: $R_2 = R_3 = 1000\ \Omega$; $C = 0.02\ \mu F$.

7.25 A Maxwell bridge, designed to measure the unknown impedance (R_u, L_u) of a coil, is shown in Figure 7.8.
(a) Explain briefly what the merits of a Maxwell bridge are compared with other forms of null-type, alternating current bridges.
(b) Derive an expression for R_u and L_u under balance conditions.
(c) If the fixed bridge component values are $R_3 = 270\ \Omega$ and $C = 50\ \mu F$, calculate the value of the unknown impedance if $R_1 = 963.9\ \Omega$ and $R_2 = 75.4\ \Omega$ at balance.
(d) Calculate the Q factor for the coil if the supply frequency is 60 Hz.

7.26 (a) For the deflection-type AC bridge shown in Figure 7.9, derive expressions for the output voltage V_0 in terms of the bridge components given for the cases of (i) Z_u and Z_1 being inductances L_u and L_1 and (ii) Z_u and Z_1 being capacitances C_u and C_1. Assume that the impedance of the instrument measuring V_0 is very high and can be neglected.
(b) The deflection-type bridge shown in Figure 7.9 is used to measure an unknown inductance L_u. If the components in the bridge have the following values: $V_s = 25 V_{rms}$, $L_1 = 10$ mH, $R_2 = 56\ \Omega$, $R_3 = 33\ \Omega$, calculate the output voltage V_0 if $L_u = 20$ mH.

7.27 Figure 7.22 shows a form of AC bridge that is commonly called a Hays bridge. This is commonly used for measuring the inductance of inductor coils where the Q factor has a high value. The inductance and resistance of the coil are represented in the figure by the symbols L_1 and R_1, respectively.
(a) Obtain the bridge balance conditions.

(b) If the Q factor for the unknown inductor in the bridge has a high value, show that the expression for the inductance value when the bridge is balanced is independent of frequency.

(c) If the Q value is high, calculate the value of the inductor if the bridge component values at balance are as follows: $R_2 = R_3 = 500\ \Omega$; $C = 0.1\ \mu F$.

7.28 (a) A deflection-type AC bridge of the form shown in Figure 7.9 is used to measure the capacitance of an unknown capacitor C_u (thus the impedances Z_u and Z_1 in the figure are replaced by capacitance values C_u and C_1). Derive an expression for the output voltage V_0 in terms of the bridge components given. Assume that the impedance of the instrument measuring V_0 is very high and can be neglected.

(b) If the components of the bridge have the following values: $V_s = 20V_{rms}$, $C_1 = 100\ \mu F$, $R_2 = 50\ \Omega$, $R_3 = 60\ \Omega$ and the bridge output voltage measured is $5V_{rms}$, calculate the value of C_u.

7.29 Discuss the alternative methods of measuring the frequency of an electrical signal and indicate the likely measurement accuracy obtained with each method.

7.30 Using the Lissajous figure method of measuring frequency, a reference frequency signal of 1 kHz is applied to the Y channel of an oscilloscope and the unknown frequency is applied to the X channel. Determine the unknown frequency for each of the oscilloscope displays shown in Figure 7.23. Also indicate the approximate phase difference of the unknown signal with respect to the reference signal.

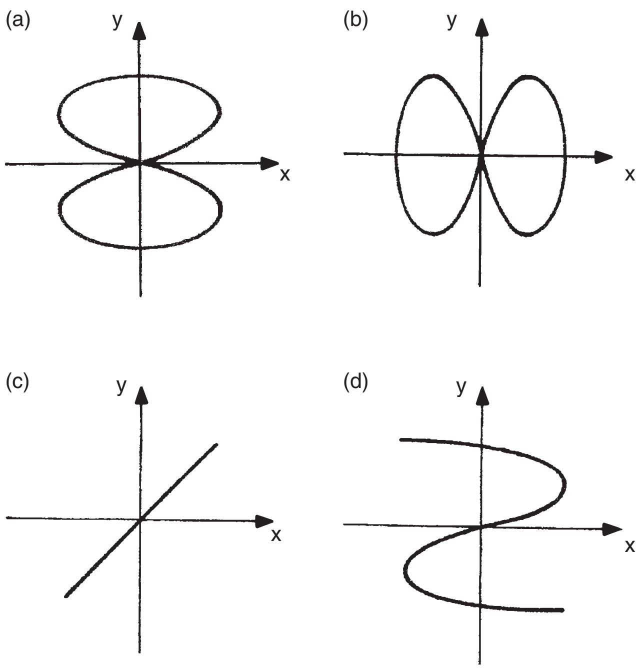

Figure 7.23
Oscilloscope displays for problem 7.30.